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Technical Note

Physical quantity synergy in laminar flow field and its application in heat transfer enhancement

Liu Wei [a,*], Liu Zhichun [a], Ming Tingzhen [a], Guo Zengyuan [b]

[a] *School of Energy and Power Engineering, Huazhong University of Science and Technology, Wuhan 430074, China*
[b] *School of Aerospace, Tsinghua University, Beijing 100084, China*

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ABSTRACT

On the basis of field synergy principle for heat transfer enhancement, physical quantity synergy in laminar flow field of convective heat transfer is analyzed according to physical nature of convective heat transfer between fluid and solid wall. Synergy regulation among physical quantities is revealed by mathematical expressions reflecting mechanism of heat transfer enhancement. Characteristic of heat transfer enhancement, which is directly associated with synergy angles α, β and γ, is also analyzed. Numerical simulation is made to verify the principle of physical quantity synergy developed in the paper.

1. Introduction

Much attention has been attracted to heat transfer enhancement for improving the overall performance of heat exchangers. Guo et al [1] afresh surveyed physical mechanism of convective heat transfer and developed field synergy principle for enhancing heat transfer. They proposed a concept that physical nature of convective heat transfer is up to synergetic relation between velocity field and heat-flux field. Under the same boundary conditions of velocity and temperature, the better the synergy between velocity field and heat-flux field is, the higher the heat transfer intensity will be. Later on, some numerical computations and experiment data verified that the field synergy principle can be used as a guide to effectively design heat transfer surfaces and heat exchangers [2–8].

2. Physical quantity synergy in non-isothermal flow field

2.1. Synergy between velocity gradient and temperature gradient

For the steady laminar heat transfer in a two-dimensional parallel channel with height H and length L, if it is analyzed symmetrically by taking channel height $h = H/2$, the energy conservation equation can be described as

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right). \tag{1}$$

If we introduce the non-dimensional numbers by

$$Y=\frac{y}{h},\quad \overline{\boldsymbol{U}}=\frac{\boldsymbol{U}}{u_m},\quad \nabla\overline{T}=\frac{\nabla T}{(T_w-T_m)/h},\quad T_w>T_m,$$

where h refers to half channel height, U refers to fluid velocity vector, u_m refers to fluid average velocity, T_w refers to channel wall temperature, T_m refers to fluid average temperature, then Eq. (1) can be rewritten as [1]

$$Nu = RePr\int_0^{\delta_t/h}(\overline{\boldsymbol{U}}\cdot\nabla\overline{\boldsymbol{T}})\boldsymbol{dY}, \tag{2}$$

where Reynolds number is $Re=\frac{u_m h}{\nu}$, Prandlt number is $Pr=\frac{\rho c_p \nu}{k}$. In fact, if the boundary layer merges in the center plane of channel, integral limit becomes $\delta_t/h = 1$, which means the fluid enters the fully developed region. So Eq. (2) can be applied in the whole channel.

In Eq. (2), dot product of non-dimensional velocity and non-dimensional temperature gradient can be expressed as [2]

$$\overline{\boldsymbol{U}}\cdot\nabla\overline{\boldsymbol{T}}=|\overline{\boldsymbol{U}}||\nabla\overline{\boldsymbol{T}}|\cos\beta. \tag{3}$$

Substituting Eq. (3) into (2), we know that dot product $\overline{\boldsymbol{U}}\cdot\nabla\overline{\boldsymbol{T}}$ increases with the decrease of synergy angle β between vectors U and ∇T. As a result, the convective heat transfer between fluid and solid wall will be enhanced. In other words, if direction of fluid velocity is more close to that of heat flux, the effect of convective heat transfer will be better in the laminar flow field.

* Corresponding author. Tel.: +86 27 8754 2618.
E-mail address: w_liu@hust.edu.cn (L. Wei).

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2.2. Synergy between velocity and velocity gradient

For the laminar flow in the two-dimensional parallel channel mentioned above, the momentum conservation equation is

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right). \tag{4}$$

Integrating Eq. (4) along y direction within boundary layer yields first, and then along x direction from channel inlet to its outlet, we obtain

$$\int_0^L\int_0^\delta \rho(\boldsymbol{U}\cdot\nabla u)dxdy=-\int_0^L\int_0^\delta\frac{\partial p}{\partial x}dxdy-\int_0^L\tau_w dx, \tag{5}$$

where τ_w stands for shear stress of wall, and integral of viscous force on channel wall is

$$\int_0^L\tau_w dx=\int_0^{L_1}\tau_{w_1}dx+\int_{L_1}^L\tau_{w_2}dx, \tag{6}$$

where τ_{w_1} and τ_{w_2} represent the shear stresses in the channel entrance region and the fully developed flow region, respectively, and can be solved as

$$\tau_{w_1}=\frac{0.323\rho u_m^2}{\sqrt{Re}\sqrt{x/h}},\quad x<L_1 \tag{7}$$

$$\tau_{w_2}=\frac{3\rho u_m^2}{Re},\quad x\geq L_1 \tag{8}$$

where L_1 stands for length of channel entrance region.

Substituting Eqs. (6)–(8) into Eq. (5), we have

$$\int_0^L\int_0^h\rho(\boldsymbol{U}\cdot\nabla u)dxdy=-\int_0^L\int_0^h\frac{\partial p}{\partial x}dxdy-\frac{0.646\rho u_m^2L_1}{\sqrt{Re}\sqrt{L_1/h}}-\frac{3\rho u_m^2(L-L_1)}{Re}. \tag{9}$$

The non-dimensional numbers are defined as

$$X=\frac{x}{L},\quad Y=\frac{y}{h},\quad \overline{\boldsymbol{U}}=\frac{\boldsymbol{U}}{\boldsymbol{u_m}},\quad \bar{\boldsymbol{u}}=\frac{\boldsymbol{u}}{\boldsymbol{u_m}},\quad \chi_1=\frac{\boldsymbol{L_1}}{\boldsymbol{L}},\quad \chi_2=\frac{\boldsymbol{L-L_1}}{\boldsymbol{L}},$$

$$\boldsymbol{Eu}=\Delta\bar{\boldsymbol{p}}=\frac{\Delta\boldsymbol{p}}{\rho\boldsymbol{u_m^2}},$$

$$\nabla\bar{\boldsymbol{u}}=\frac{\left(\frac{\partial}{\partial\boldsymbol{x}}\boldsymbol{i}+\frac{\partial}{\partial\boldsymbol{y}}\boldsymbol{j}\right)\boldsymbol{u}}{\boldsymbol{u_m}/\boldsymbol{h}},\quad \nabla\bar{\boldsymbol{p}}=\frac{\left(\frac{\partial}{\partial\boldsymbol{x}}\boldsymbol{i}+\frac{\partial}{\partial\boldsymbol{y}}\boldsymbol{j}\right)\boldsymbol{p}}{\rho\boldsymbol{u_m^2}/\boldsymbol{h}},\quad \frac{\partial}{\partial\boldsymbol{y}}\boldsymbol{j}=\boldsymbol{0},$$

where Eu refers to Euler number, Δp refers to pressure drop between channel inlet and outlet, $\boldsymbol{i}$ and $\boldsymbol{j}$ refer to unit vectors of x and y coordinates, respectively.

Then Eq. (9) can be rewritten as

$$\int_0^1\int_0^{\delta/h}(\overline{\boldsymbol{U}}\cdot\nabla\bar{\boldsymbol{u}})\boldsymbol{dXdY}=-\int_{\boldsymbol{0}}^{\boldsymbol{1}}\int_{\boldsymbol{0}}^{\delta/\boldsymbol{h}}(\nabla\bar{\boldsymbol{p}}\cdot\boldsymbol{I})\boldsymbol{dXdY}-\frac{\boldsymbol{0.646}\chi_1}{\sqrt{\boldsymbol{Re}}\sqrt{\boldsymbol{L_1/h}}}-\frac{\boldsymbol{3}\chi_2}{\boldsymbol{Re}}, \tag{10}$$

where δ/h refers to non-dimensional thickness of velocity boundary layer. If the velocity boundary layer merges in the center plane of channel, integral limit becomes $\delta/h = 1$, which means the channel flow becomes fully developed. Integral term on the right-hand side refers to non-dimensional pressure drop for the parallel channel with height $h = H/2$, and can be expressed as

$$\Delta\bar{p}=-\int_0^1\int_0^{\delta/h}(\nabla\bar{p}\cdot\boldsymbol{I})dXdY, \tag{11}$$

where $\boldsymbol{I}$ refers to unit vector.

From Eqs. (10) and (11), we can have

$$Eu=\frac{0.646\chi_1}{\sqrt{Re}\sqrt{L_1/h}}+\frac{3\chi_2}{Re}+\int_0^1\int_0^{\delta/h}(\overline{\boldsymbol{U}}\cdot\nabla\bar{\boldsymbol{u}})\boldsymbol{dXdY}, \tag{12}$$

where dot product of non-dimensional velocity and non-dimensional velocity gradient can be expressed as

$$\overline{\boldsymbol{U}}\cdot\nabla\bar{\boldsymbol{u}}=|\overline{\boldsymbol{U}}||\nabla\bar{\boldsymbol{u}}|\cos\alpha. \tag{13}$$

Substituting Eq. (13) into (12), we find that dot product $\overline{\boldsymbol{U}}\cdot\nabla\bar{\boldsymbol{u}}$ decreases with the increase of synergy angle α between vectors U and ∇u. As a result, the flow resistance of fluid will decrease. If synergy angle α in all cross sections of parallel channel is kept as 90°, we have $\overline{\boldsymbol{U}}\cdot\nabla\bar{\boldsymbol{u}}\equiv\boldsymbol{0}$. Then the expression of pressure drop of parallel channel with length L and height $H/2$ becomes

$$\Delta p_L=\Delta\bar{p}L=\frac{0.646\chi_1 L}{\sqrt{Re}\sqrt{L_1/h}}+\frac{3\chi_2 L}{Re}. \tag{14}$$

2.3. Synergy among velocity, temperature gradient and velocity gradient

Fig. 1 shows synergy relation among velocity, temperature gradient and velocity gradient of a fluid particle M in the laminar flow field. Three correlated synergy angles are

$$\alpha=\arccos\frac{\boldsymbol{U}\cdot\nabla u}{|\boldsymbol{U}||\nabla u|}, \tag{15}$$

$$\beta=\arccos\frac{\boldsymbol{U}\cdot\nabla T}{|\boldsymbol{U}||\nabla T|}, \tag{16}$$

$$\gamma=\arccos\frac{\nabla T\cdot\nabla u}{|\nabla T||\nabla u|}, \tag{17}$$

where vectors U, ∇T and ∇u are coplanar in the two-dimensional laminar flow field. Thus all fluid particles on a stream line satisfy with $\gamma\equiv|\alpha-\beta|$, and this equality holds strictly for the laminar heat transfer in parallel channel or bare tube.

In order to raise the overall performance of a heat transfer unit, the synergy between temperature gradient ∇T and velocity gradient ∇u should be considered. The bigger the synergy angle γ is, the higher the PEC value will be. The PEC is an evaluation coefficient representing comprehensive performance of a heat transfer unit, which is commonly given as

$$PEC=\frac{Nu/Nu_0}{(f/f_0)^{1/3}}, \tag{18}$$

where Nu_0 and f_0 stand for Nusselt number and fluid resistance coefficient in parallel channel or bare tube, respectively.

It can be found from Fig. 1 that there are three ways of changing synergetic relation between ∇T and ∇u to enlarge synergy angle γ and raise PEC value. (1) Angle α remains constant while angle β decreases, then angle γ and PEC value will increase; (2) angle β remains constant while angle α increases, then angle γ and PEC

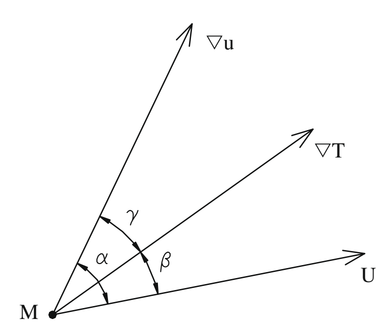

Fig. 1. Synergy relation among velocity, temperature gradient and velocity gradient of a fluid particle M in laminar flow field.

value will increase; (3) angle β decreases while angle α increases, then angle γ and *PEC* value will increase. In the third case, not only heat transfer is enhanced, but also flow resistance is reduced, therefore comprehensive performance of a heat transfer unit will be raised.

3. Numerical verification for physical quantity synergy

3.1. Physical model

As shown in Fig. 2, in a two-dimensional parallel channel with height 20 mm and length 500 mm, two cylinders with diameter 6 mm are interpolated along its flow direction with an interval of 20 mm. A parallel channel with length 1500 mm is settled in the upstream of cylinder-interpolated parallel channel to allow fluid to become fully developed. Based on this model, the numerical verification for the principle of physical quantity synergy in the non-isothermal flow field is carried out.

3.2. Results and discussion

To solve the equations in the above model, finite difference method and two-order upwind difference scheme are applied in numerical computation, and SIMPLE algorithm is used for coupling of pressure and velocity. The boundary values for numerical simulation are setting as: wall temperature of parallel channel $T_w = 350$ K, inlet temperature of fluid $T_\infty = 293$ K. The computational fluid is water and its physical properties are kept as constant. The computational results are illustrated in Figs. 3–6.

Fig. 3 shows the effect of *Re* number on average synergy angle β for parallel channel and cylinder-interpolated parallel channel. As shown in the figure, average synergy angle β between fluid velocity U and temperature gradient ∇T in cylinder-interpolated parallel channel is smaller than that in parallel channel, so it can be inferred from Eq. (2) that heat transfer will be enhanced between fluid and wall. Fig. 4 shows the effect of *Re* number on average synergy angle α for parallel channel and cylinder-interpolated parallel channel. As shown in the figure, average synergy angle α between fluid velocity U and velocity gradient ∇u in cylinder-interpolated parallel channel is smaller than that in parallel channel, so it can be found from Eq. (12) that flow resistance of fluid will increase. Fig. 5 shows the relation between *Re* number and average synergy angle γ for parallel channel and cylinder-interpolated parallel channel. As shown in the figure, average synergy angle γ between temperature gradient ∇T and velocity gradient ∇u in cylinder-interpolated parallel channel is bigger than that in parallel channel, so the performance of heat transfer unit will be improved by interpolating cylinders due to disturbing fluid.

Fig. 6 indicates the relation between *Re* number and *PEC* value of cylinder-interpolated parallel channel. For the scope of *Re* number in the figure, the *PEC* value of cylinder-interpolated parallel channel falls into the range of 0.64–0.78. Even though the *PEC* value increases with the increase of *Re* number, the comprehensive performance of the channel is not ideal in this particular case. So synergy angle γ should be enlarged to raise the *PEC* value.

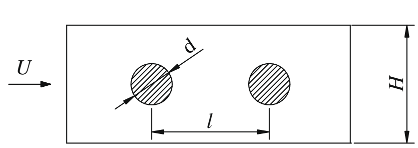

Fig. 2. Physical model of cylinder-interpolated parallel channel.

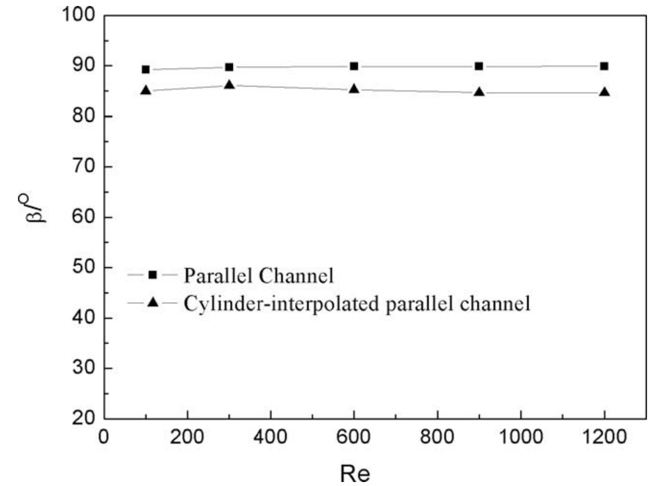

Fig. 3. Relation between *Re* number and average synergy angle β in parallel channel and cylinder-interpolated parallel channel.

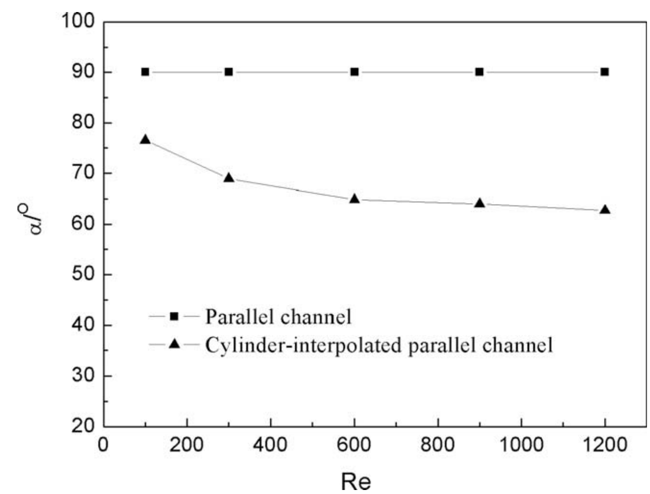

Fig. 4. Relation between *Re* number and average synergy angle α in parallel channel and cylinder-interpolated parallel channel.

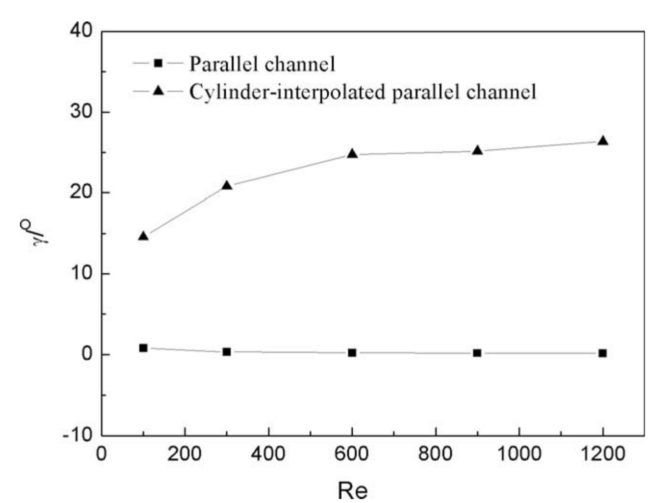

Fig. 5. Relation between *Re* number and average synergy angle γ in parallel channel and cylinder-interpolated parallel channel.

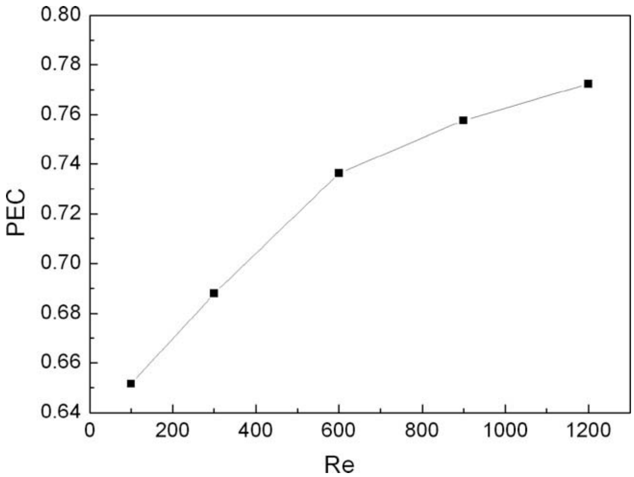

Fig. 6. Relation between *PEC* of cylinder-interpolated parallel channel and *Re* number.

4. Conclusions

(1) For convective heat transfer in the laminar flow field, there exists synergetic relation among physical quantities in which the relation among vectors is direct, and the relation among scalars is indirect. Improving synergetic relation among physical quantities is beneficial to heat transfer enhancement, which is related to the change of vector directions in the non-isothermal flow field.

(2) Synergy angles α, β and γ indicates the extent to which heat transfer is enhanced, flow resistance is reduced, and comprehensive performance is improved. Fluid resistance coefficient f decreases with the increase of synergy angle α, convective heat transfer coefficient h increases with the decrease of synergy angle β, and performance evaluation coefficient *PEC* increases with the increase of synergy angle γ. Those may guide the design for heat transfer units or heat exchangers.

Acknowledgments

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